Imperial College London

Solution techniques for large non-linear vibration

Loïc Salles

10 January 2018

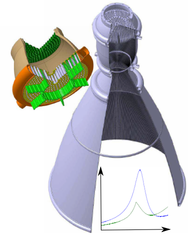
Outline

The lecture consists of three parts:

1. Presentation of our in-house software FORSE
2. Introduction to time spectral methods
3. Introduction to continuation techniques

Some words about me

- 2002-2006 Engineering study at Ecole Centrale Lyon, France
- 2004-2006 Msc at BMSTU and Keldysh Research Center for Rocket engines: *High Frequency instability in combustion chamber of a liquid rocket engine*



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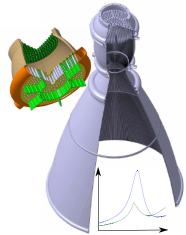

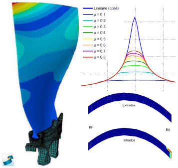

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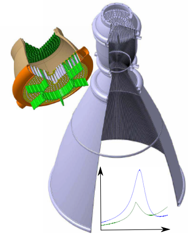
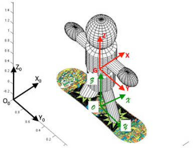
- 2012-now: Researcher at Rolls-Royce VUTC Imperial College



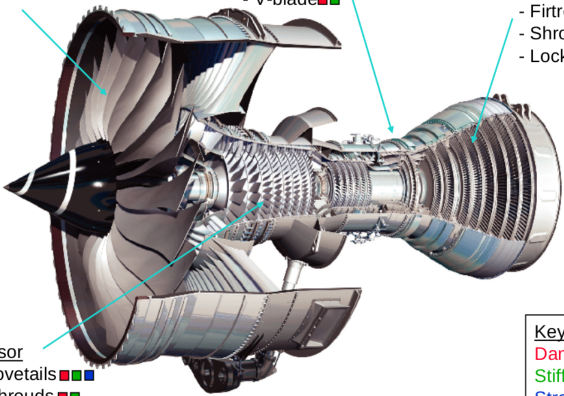


Joints in aeroengine

Fan
- dovetail
- snubbers

Engine Structure
- Flanges / spigots
- Splines
- V-blade

Turbine Blades
- Dampers / seals
- Firtree
- Shroud
- Lockplate / cover plates

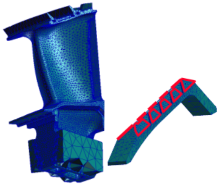


Compressor
- Rotor dovetails
- Stator shrouds

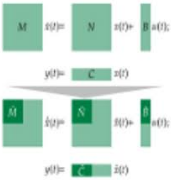
Key
- Damping
- Stiffness / frequency
- Stress

Components of the analysis

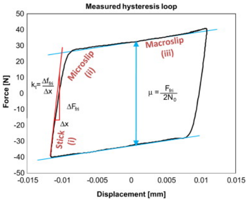
Geometry



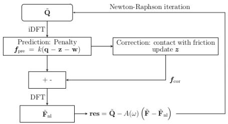
ROM



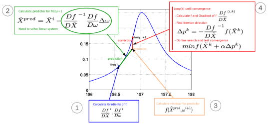
Contact



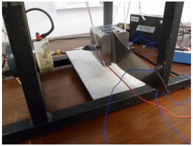
HBM-AFT



Continuation

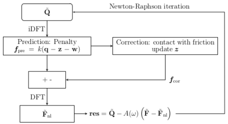


Validation

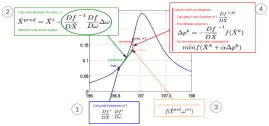


Components of the analysis

HBM-AFT



Continuation



FORSE

FORSE (which stands for FOrced Response SuitE) is the program developed at the Imperial College Vibration UTC since 2000

1. multi-Harmonic forced response of nonlinear (and linear, as a particular case) mechanical systems which are subjected to periodic excitation
2. sensitivity of the forced response to variation of selected design parameters
3. dependency of the forced response levels and resonance frequencies on parameters of contact interfaces and some other design parameters

Features FORSE

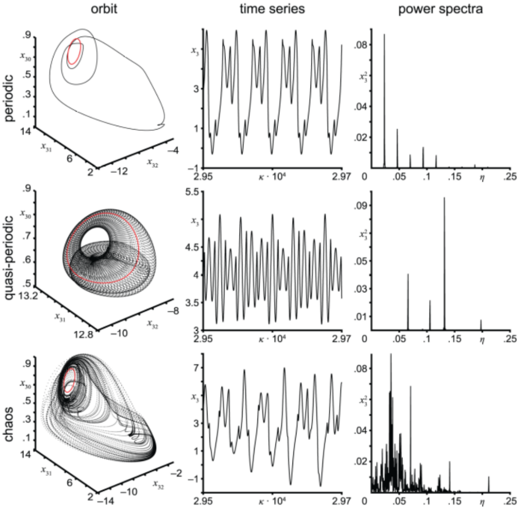
- Frequency domain analysis based on Fourier series

- Loading and Solution
  - Quasi-static
  - Periodic loading
  - Frequency dependant loading
  - Nonlinear Modal Analysis
  - Modal Analysis for Flutter
- Modelling
  - Inertia effect
  - Gyroscopic effect
  - Centrifugal force
  - Viscous and hysteresis damping
  - Modal forces
- Cyclic symmetry
  - Cyclic boundaries
  - Nonlinear Cyclic boundaries
  - Mistuning:
    - Linear analysis
    - Nonlinear analysis
- Non-linearities
  - Friction 1D,2D,3D
  - Gap element
  - Rubbing element
  - polynomial,piecewise polynomial
  - User defined (plugin)

Features of FORSE

- Type of analysis
  - Parameter continuation with fixed frequency
  - Frequency Forced response
  - Resonance versus parameter variation
  - Frequency displacement response
  - Limit cycle oscillation calculation
- Analysis of obtained solution
  - Sensitivity analysis 1st and 2nd order
  - Stability analysis
  - Bifurcation detection
  - Branch following

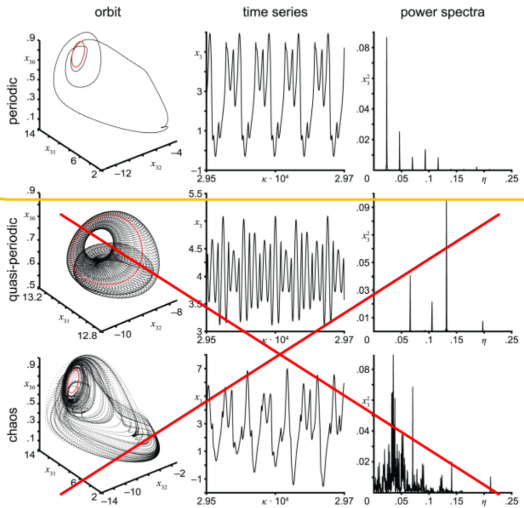
Type of Response



Type of Response

FORSE

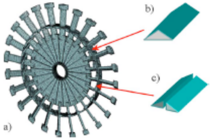
FORSE: Detection by stability analysis

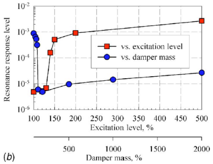


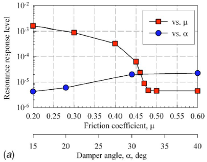
Coding aspect

- FORTRAN 2008 OBJECT ORIENTED PROGRAMMING
- Orthogonalisation of the code
- Each nonlinear element is a class
- Parallel coding using OPENMP (MPI in progress)
- Possibility to use also the free Lapack iibrary
- In-house nonlinear solver: Newton-Raphson and Free Jacobian method
- Direct and Iterative linear solvers
    - LU decomposition
    - GMRES solver
- Linked to (MKL) BLAS, LAPACK and PETSC
- Compiled with Intel, GNU and PGI compilers

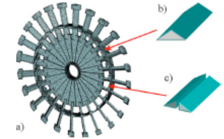
Friction damper

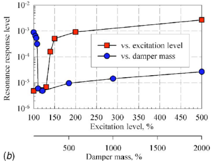


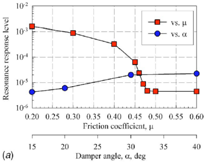


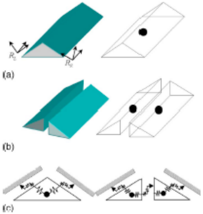


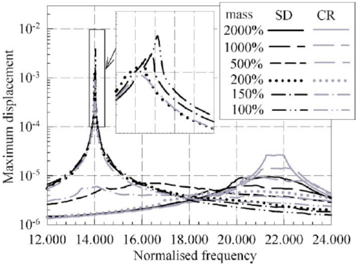
Friction damper

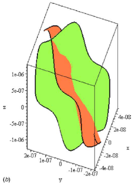




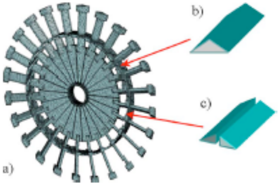


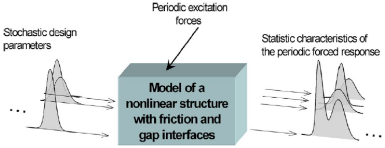


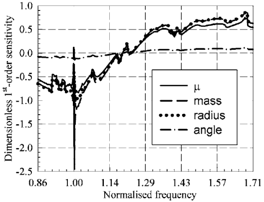


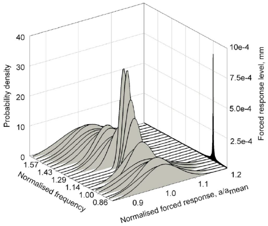


Sensitivity Analysis

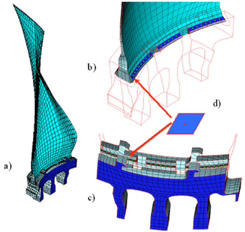


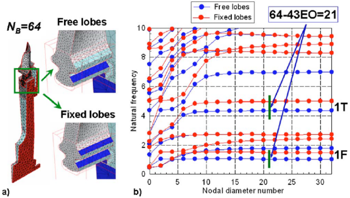


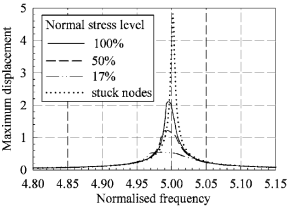


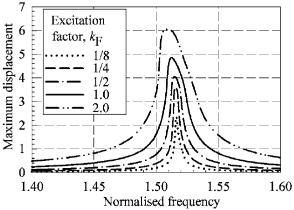


Root damping

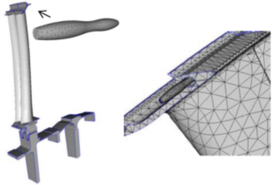


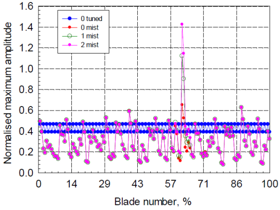


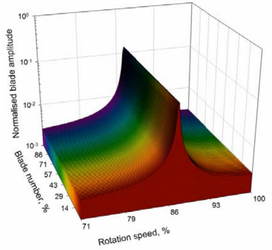


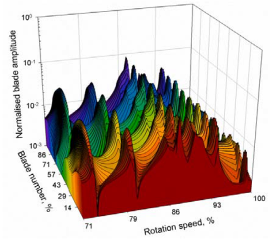


Mistuning: loss of damper

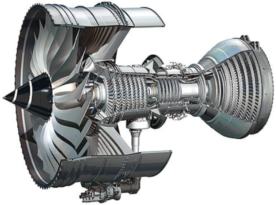


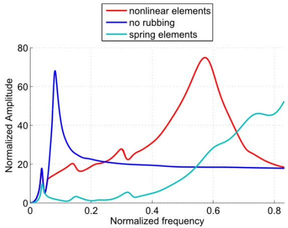


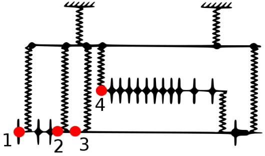


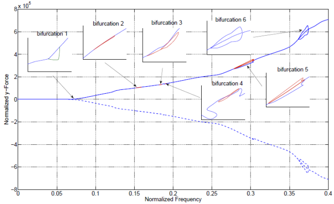


Whole Engine Modelling



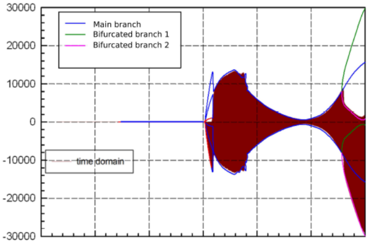




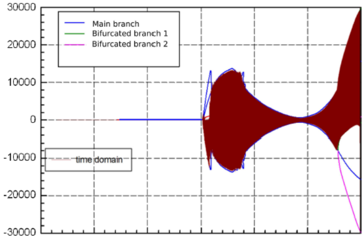


Whole Engine Modelling

acceleration



deceleration

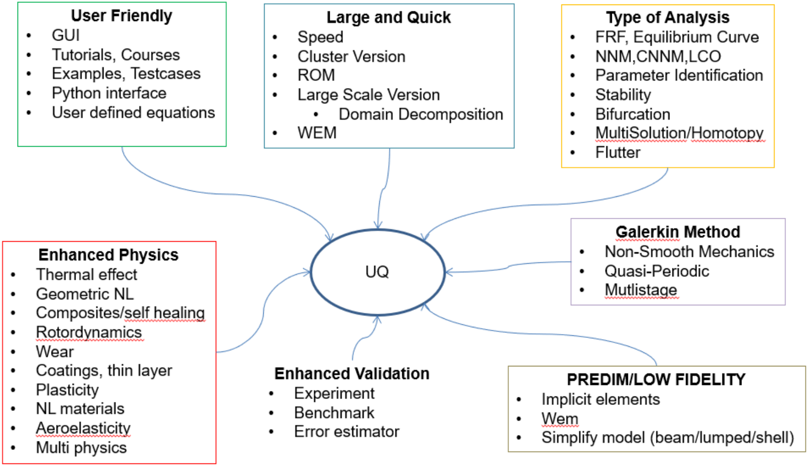


Good agreement between harmonic balance method and transient simulation

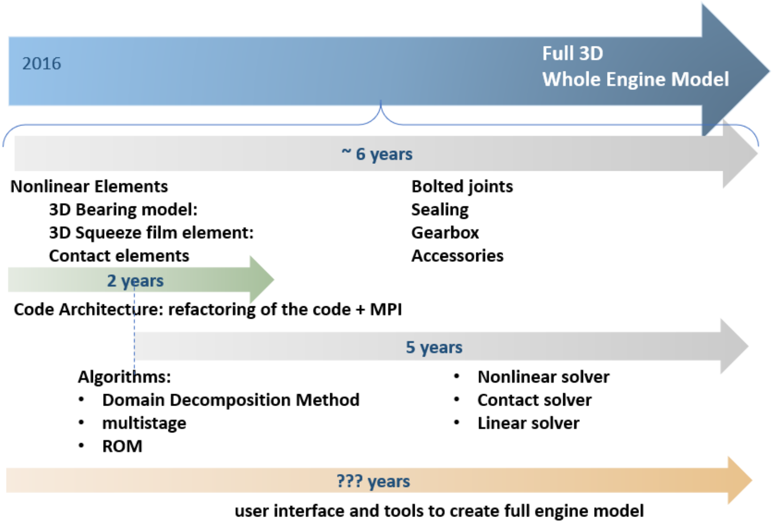
CPU time:

- HBM: 20 minutes
- Transient: 24 hours

Brainstorming



Roadmap WEM

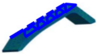
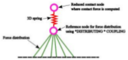
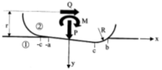
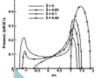

2016
Full 3D
Whole Engine Model
~ 6 years
Nonlinear Elements
3D Bearing model:
3D Squeeze film element:
Contact elements
Bolted joints
Sealing
Gearbox
Accessories
2 years
Code Architecture: refactoring of the code + MPI
5 years
Algorithms:
Domain Decomposition Method
multistage
ROM
Nonlinear solver
Contact solver
Linear solver
??? years
user interface and tools to create full engine model

Contact modelling

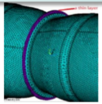
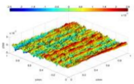
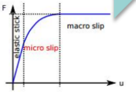
Implicit element:

- 6 DOFs damper
- RBE3 Reduced Order Modelling

- Microslip model with normal force law
- Semi Analytical method

2016 → **Improve joint modelling** → **Accurate modelling of joints**

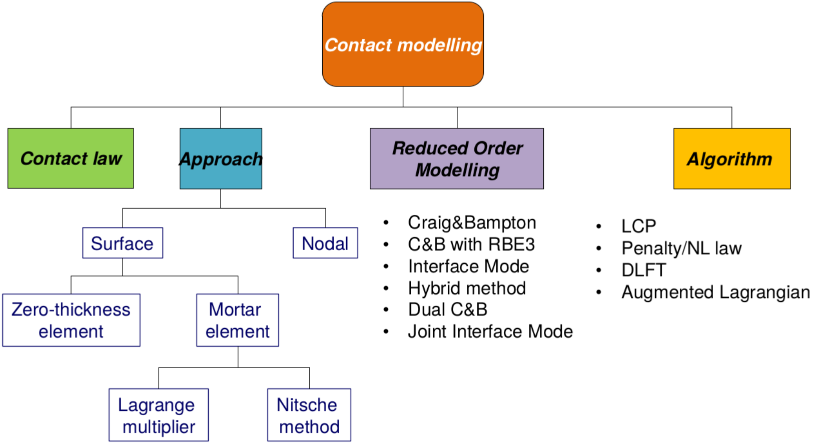




Explicit element:

- FE formulation of the contact:
  - Zero thickness element
  - Mortar method

- Microscale effect
  - Roughness
  - Dry lubricant
  - Fretting-wear
- New reduced order modelling
- *A priori* and *a posteriori* error estimator

Contact modelling



Introduction to Spectral Methods for Periodic Problems

Loïc Salles

Munich

January 2018

Outline

Outline

Equation of motion

Equation of motion

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{C}\dot{\boldsymbol{U}} + \boldsymbol{K}\left(\boldsymbol{U},\Omega\right)\boldsymbol{U} + \boldsymbol{F}_c(\boldsymbol{U},\dot{\boldsymbol{U}}) = \boldsymbol{F}_{ex}(t) \tag{1}$$

with periodic condition

$$\boldsymbol{U}(0) = \boldsymbol{U}(T) \quad \dot{\boldsymbol{U}}(0) = \dot{\boldsymbol{U}}(T) \tag{2}$$

List of methods

- Numerical integration with transient response

List of methods

- Numerical integration with transient response
- Shooting method

List of methods

- Numerical integration with transient response
- Shooting method
- (pseudo)-Spectral methods
  - Fourier Series
  - Polynomial series
  - Wavelet Galerkin method

List of methods

- Numerical integration with transient response
- Shooting method
- (pseudo)-Spectral methods
  - Fourier Series
  - Polynomial series
  - Wavelet Galerkin method
- Finite Element in Time

Outline

Fourier Galerkin Method

$\boldsymbol{U}$ can be written as a Fourier series:

$$\boldsymbol{U}(\tau) = \tilde{\boldsymbol{U}}_0 + \sum_{n=1}^{Nh} \tilde{\boldsymbol{U}}_{n,c} \cos(n\tau) + \tilde{\boldsymbol{U}}_{n,s} \sin(n\tau) \quad (3)$$

where Nh is the number of temporal harmonics retained and $\tau = \omega t$ is the normalized time of the vibration period.

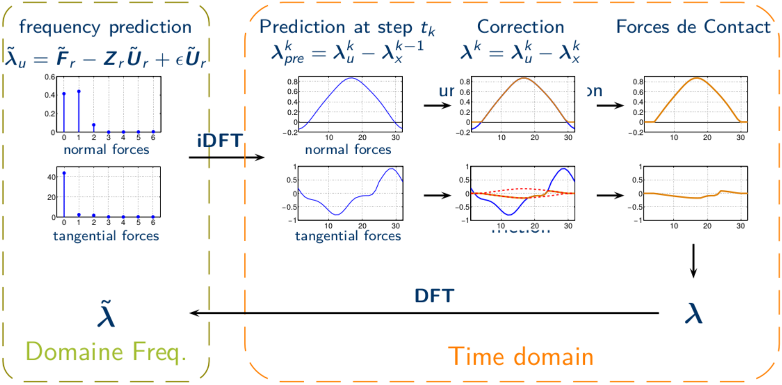
$$\boldsymbol{Z}\tilde{\boldsymbol{U}} + \tilde{\boldsymbol{F}}_c = \tilde{\boldsymbol{F}}_{ex} \quad (4)$$

$\boldsymbol{Z}$ is the dynamical stiffness

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{K} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{K} - (N_h\omega)^2\boldsymbol{M} & N_h\omega\boldsymbol{C} \\ \mathbf{0} & \mathbf{0} & -N_h\omega\boldsymbol{C} & \boldsymbol{K} - (N_h\omega)^2\boldsymbol{M} \end{bmatrix} \quad (5)$$

AFT procedure with contact

Alternate frequency time procedure - calculation of contact forces



Discrete Fourier Transform

$$\boldsymbol{U}(\tau_k) = \bar{\boldsymbol{U}}_k = \tilde{\boldsymbol{U}}_0 + \sum_{n=1}^{Nh} \tilde{\boldsymbol{U}}_{n,c} \cos(n\tau_k) + \tilde{\boldsymbol{U}}_{n,s} \sin(n\tau_k).$$ Definition of an operator

$$\bar{\boldsymbol{U}} = \boldsymbol{T}^{-1}\tilde{\boldsymbol{U}} \qquad (6)$$

where $\boldsymbol{T}$ is a matrix of the discrete Fourier transform whose size is equal to $(2Nh+1)Q$ by $(2Nh+1)Q$.

$$\boldsymbol{T} = \frac{2}{2Nh+1} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \dots & \frac{1}{2} \\ \cos\tau_0 & \cos\tau_1 & \dots & \cos\tau_{2*Nh+1} \\ \sin\tau_0 & \sin\tau_1 & \dots & \sin\tau_{2*Nh+1} \\ \dots & & & \\ \cos Nh\tau_0 & \cos Nh\tau_1 & \dots & \cos Nh\tau_{2*Nh+1} \\ \sin Nh\tau_0 & \sin Nh\tau_1 & \dots & \sin Nh\tau_{2*Nh+1} \end{bmatrix} \otimes \boldsymbol{I}_N \qquad (7)$$

Trigonometric collocation

Equation of motion at each collocation point (time step τ_k)

$$f(\tilde{\boldsymbol{U}}) = \begin{cases} \boldsymbol{M}\ddot{\boldsymbol{U}}(\tau_1) + \boldsymbol{C}\dot{\boldsymbol{U}}(\tau_1) + \boldsymbol{K}\boldsymbol{U}(\tau_1) + \boldsymbol{F}_c(\tau_1) - \boldsymbol{F}_{ex}(\tau_1) \\ \dots \\ \boldsymbol{M}\ddot{\boldsymbol{U}}(\tau_k) + \boldsymbol{C}\dot{\boldsymbol{U}}(\tau_k) + \boldsymbol{K}\boldsymbol{U}(\tau_k) + \boldsymbol{F}_c(\tau_k) - \boldsymbol{F}_{ex}(\tau_k) \\ \dots \\ \boldsymbol{M}\ddot{\boldsymbol{U}}(\tau_{2Nh+1}) + \boldsymbol{C}\dot{\boldsymbol{U}}(\tau_{2Nh+1}) + \boldsymbol{K}\boldsymbol{U}(\tau_{2Nh+1}) + \\ \qquad \boldsymbol{F}_c(\tau_{2Nh+1}) - \boldsymbol{F}_{ex}(\tau_{2Nh+1}) \end{cases} \tag{8}$$

Let use the dynamic stiffness matrix of the HBM, the system is equivalent to:

$$f(\tilde{\boldsymbol{U}}) = \boldsymbol{T}^{-1}\boldsymbol{Z}_r\tilde{\boldsymbol{U}} + \bar{\boldsymbol{F}}_c - \bar{\boldsymbol{F}}_r \tag{9}$$

Time Spectral Method

The Fourier coefficients and time variables retained are linked by:

$$\tilde{\boldsymbol{U}} = \boldsymbol{T}\bar{\boldsymbol{U}} \quad \text{and} \quad \bar{\boldsymbol{U}} = \boldsymbol{T}^{-1}\tilde{\boldsymbol{U}} \tag{10}$$

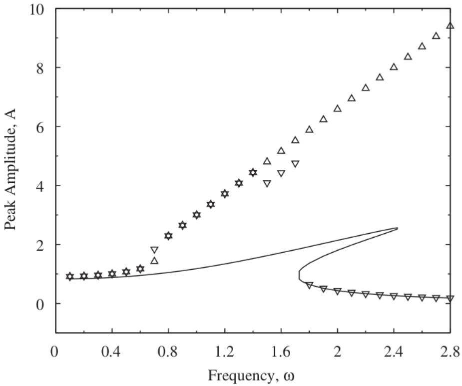
By introducing expressions of Eq. (10) in Eq. (4) the following non-linear system is obtained:

$$\boldsymbol{H}_r\bar{\boldsymbol{U}} + \bar{\boldsymbol{F}}_c = \bar{\boldsymbol{F}}_r \tag{11}$$

where $\boldsymbol{H}_r = \boldsymbol{T}\boldsymbol{Z}_r\boldsymbol{T}^{-1}$, this matrix is full.

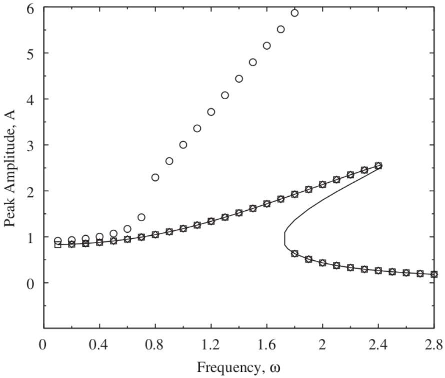
Aliasing

Duffing example $\ddot{x} + c\dot{x} + kx + \gamma x^3 = fsin(\omega t)$



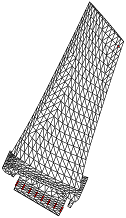
Aliasing

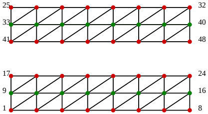
Duffing example $\ddot{x} + c\dot{x} + kx + \gamma x^3 = fsin(\omega t)$

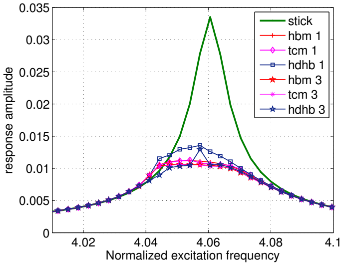


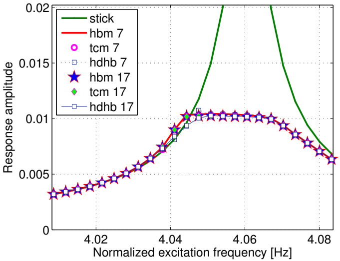
with filtering high frequencies

Examples

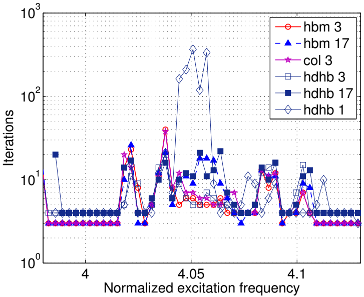



25
33
41
32
40
48
17
9
1
24
16
8


stick
hbm 1
tcm 1
hdhb 1
hbm 3
tcm 3
hdhb 3
response amplitude
Normalized excitation frequency
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
4.02
4.04
4.06
4.08
4.1


stick
hbm 7
tcm 7
hdhb 7
hbm 17
tcm 17
hdhb 17
Response amplitude
Normalized excitation frequency [Hz]
0.02
0.015
0.01
0.005
0
4.02
4.04
4.06
4.08

Comparison



	HBM		TCM		HDHB
Nh	$nit = 105$	$nit = 2Nh + 1$	$nit = 105$	$nit = 2Nh + 1$	$nit = 2Nh + 1$
1	1	0.17	1.43	0.18	0.17
3	3.71	0.51	2.70	0.45	0.71
7	14.59	6.41	7.10	3.24	3.71
17	521	88.44	44.62	46.60	46.70

Table: CPU time necessary for calculating 50 frequencies

Outline

Time Finite Element Method

Hamilton's Weak Principle:

$$\int_{t_0}^{t_F} (\delta \mathcal{L} + \delta \mathcal{W}) dt = \left[\delta q \frac{\delta \mathcal{L}}{\delta \dot{q}} \right]_{t_0}^{t_F}$$

Time Finite Element Method

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Generic Dynamical System:

$$\int_{t_0}^{t_F} (\delta\dot{x}M\dot{x} + \delta x(-C\dot{x} - Kx + F_{NL}(x,\dot{x},t))dt = [\delta x \cdot M\dot{x}]_{t_0}^{t_F}$$

Time Finite Element Method

Hamilton's Weak Principle:

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Time Discretisation: $x(t) = \sum X_i N_i(t)$ $\dot{x}(t) = \sum X_i \dot{N}_i(t)$

Time Finite Element Method

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Time Discretisation: $x(t) = \sum X_i N_i(t)$ $\dot{x}(t) = \sum X_i \dot{N}_i(t)$

Assembled System:

$$\mathbf{A}\hat{\mathbf{X}} = \hat{\mathbf{F}}_{\mathbf{NL}}(X) + \hat{\mathbf{B}}$$

with:

$$\mathbf{A} = \mathbf{M} \otimes \mathbf{L_{2,t}} + \mathbf{C} \otimes \mathbf{L_{1,t}} + \mathbf{K} \otimes \mathbf{L_{0,t}}$$

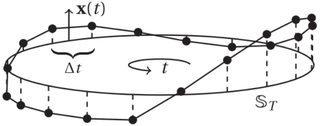
Same Form as HBM:

Galerkin Methods

$\hat{\mathbf{F}}_{\mathbf{NL}}(X,t)$ already discretised in time

Time Finite Element Method

Enforcing the **periodicity conditions**:



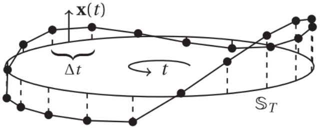
$$\mathbf{L_{0,t}} = \frac{1}{6}\begin{bmatrix} 4 & 1 & \cdots & 0 & 1 \\ 1 & 4 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 4 & 1 \\ 1 & 0 & \cdots & 4 & 1 \end{bmatrix}$$

$$\mathbf{L_{1,t}} = \frac{1}{\Delta t}\begin{bmatrix} 1 & 0 & \cdots & 0 & -1 \\ -1 & 1 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 1 & 0 \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$$

$$\mathbf{L_{2,t}} = \frac{1}{2\Delta t^2}\begin{bmatrix} 0 & -1 & \cdots & 0 & 1 \\ 1 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 0 & -1 \\ -1 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

Time Finite Element Method

Enforcing the **periodicity conditions**:



$$\mathbf{L_{0,t}} = \frac{1}{6}\begin{bmatrix} 4 & 1 & \cdots & 0 & 1 \\ 1 & 4 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 4 & 1 \\ 1 & 0 & \cdots & 4 & 1 \end{bmatrix}$$

$$\mathbf{L_{1,t}} = \frac{1}{\Delta t}\begin{bmatrix} 1 & 0 & \cdots & 0 & -1 \\ -1 & 1 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 1 & 0 \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$$

$$\mathbf{L_{2,t}} = \frac{1}{2\Delta t^2}\begin{bmatrix} 0 & -1 & \cdots & 0 & 1 \\ 1 & 0 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 0 & -1 \\ -1 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

The **flux term** becomes:

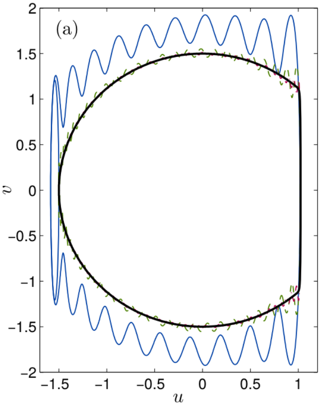
$$\hat{B} = \{\mathrm{B}_0 + \mathrm{B}_F, 0, \cdots, 0, 0\} = \{0, \cdots, 0\}$$

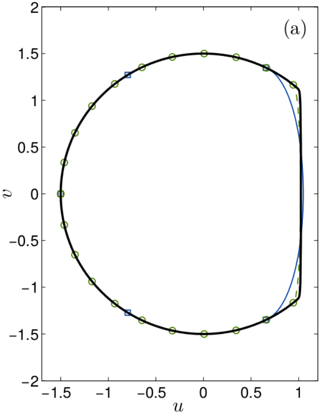
And the algebraic system:

Outline

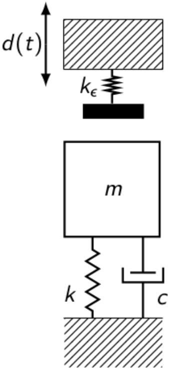
Gibbs phenomenon

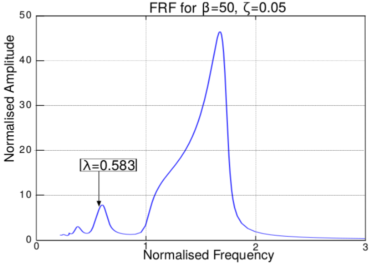
$$\ddot{u} + u + e^{(\alpha(u-1))} = 0$$



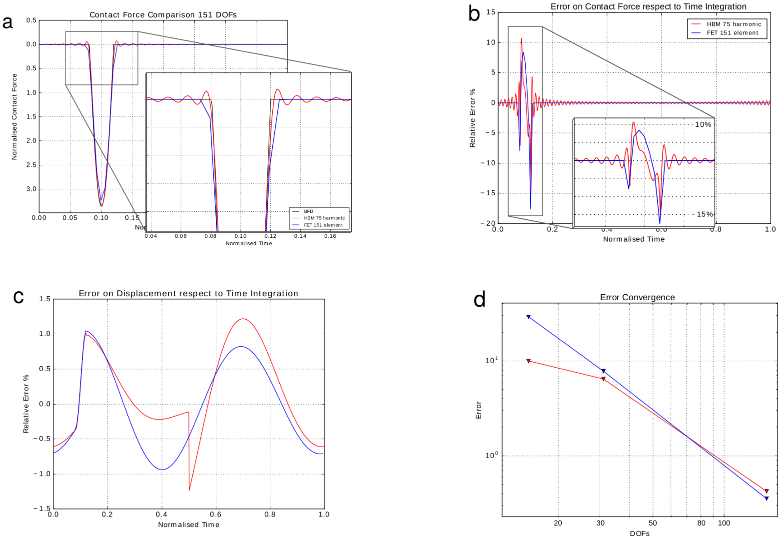


Gibbs phenomenon





Gibbs phenomenon



Outline

Avaialble code

- MANLAB http://manlab.lma.cnrs-mrs.fr/
- PyMAN https://bitbucket.org/vinus23/pyman
- Xyce https://xyce.sandia.gov/
- AUTO http://indy.cs.concordia.ca/auto/
- FE package
  - MSC Nastran 2016 (rotorsynamics and very basic)
  - cast3m http://www-cast3m.cea.fr/
  - Code Aster (only for Nonlinear Normal Modes)

Imperial College London

Introduction to Continuation Methods

Loïc Salles

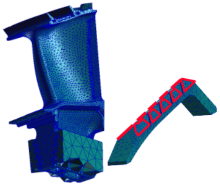
10 January 2018

Outline

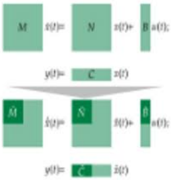
Outline

Components of the analysis

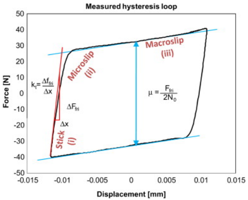
Geometry



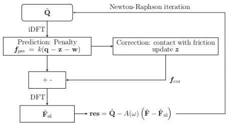
ROM



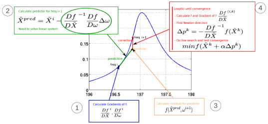
Contact



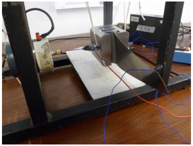
HBM-AFT



Continuation

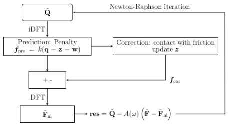


Validation

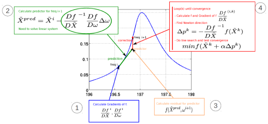


Components of the analysis

HBM-AFT



Continuation



Residual

Nonlinear system depending on one parameter

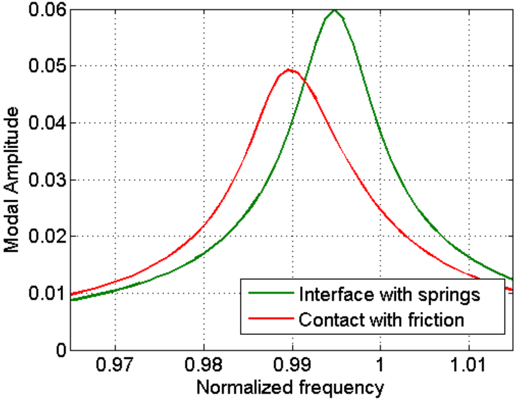
$$\boldsymbol{R}(\boldsymbol{X}, \lambda) = \mathbf{0} \tag{1}$$

- Newtons method for solving a nonlinear equation (1) may not converge if the " initial guess " is not close to a solution.
- The Implicit Function Theorem insure that the path can be followed w.r.t the parameter λ.

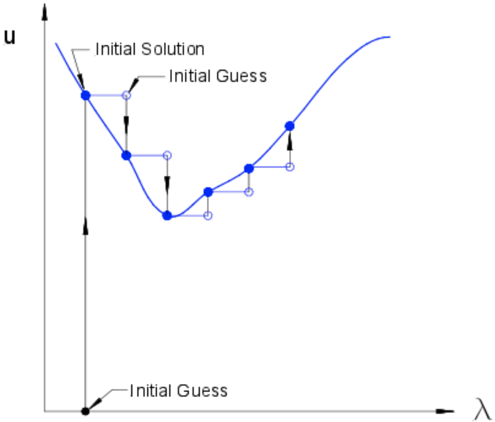
List of methods

- Predictor-Corrector Method
- Asymptotic Numerical Method (MAN)
- Homotopy Method
- Cell mapping
- ...

Building a FRF

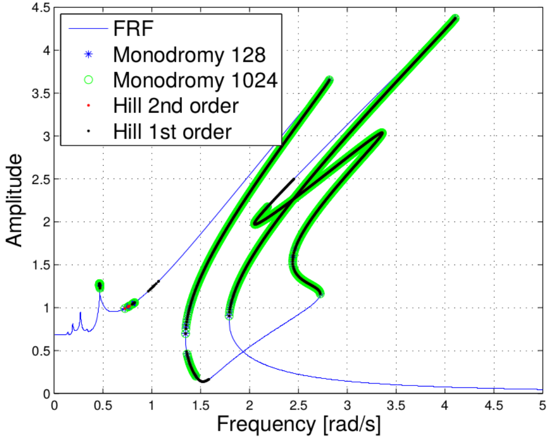


Natural Continuation



https://en.wikipedia.org/wiki/Numerical_continuation

Building non-linear FRF



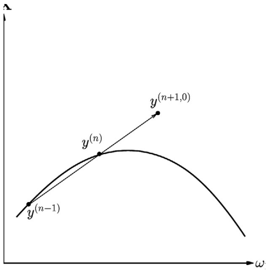
Outline

Predictors

- Constant predictor

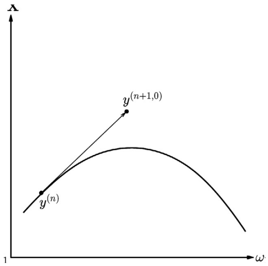
Predictors

- Constant predictor
- Secant predictor



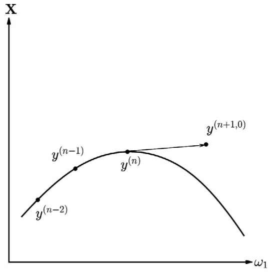
Predictors

- Constant predictor
- Secant predictor
- Tangent predictor



Predictors

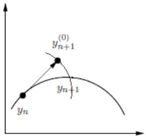
- Constant predictor
- Secant predictor
- Tangent predictor
- High-order predictor: Lagrange interpolation, spline, . . .



Correctors

arc-length method (Crisfield)

$$G(y, \Delta s) = \begin{cases} R(\boldsymbol{X}, \lambda) = 0 \\ \|\Delta \boldsymbol{X}\|^2 + \Delta\lambda^2 - \Delta s^2 = 0 \end{cases}$$



Correctors

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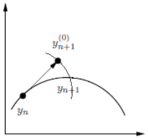
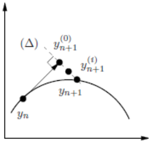


pseudo-arc-length (Riks)

$$G(y, \Delta s) = \begin{cases} R(\boldsymbol{X}, \lambda) = 0 \\ D_y R \Delta \boldsymbol{y} - \Delta s = 0 \end{cases}$$



Correctors

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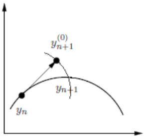

pseudo-arc-length (Riks)

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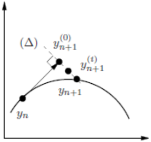
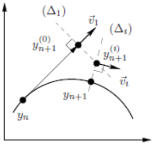
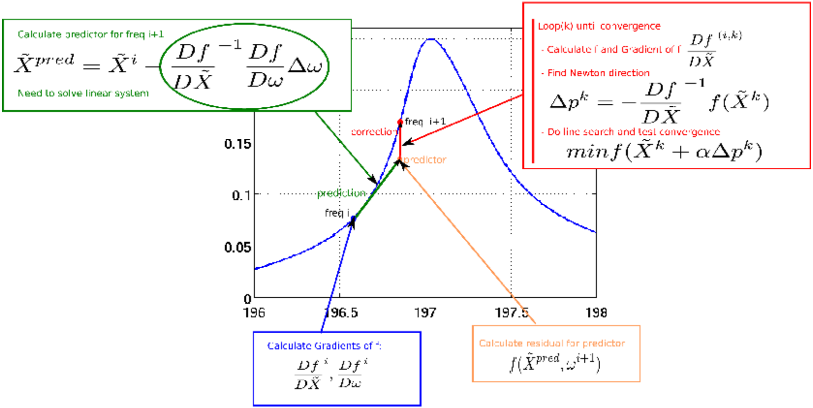


Gauss-Newton (pseudo-inverse, Moore-Penrose, Fried)

$$y_{n+1}^{j+1} = y_{n+1}^{j} - D_y R(y_{n+1}^{j})^{+} R(y_{n+1}^{j})$$



Building a FRF



Outline

Non-Linear solver

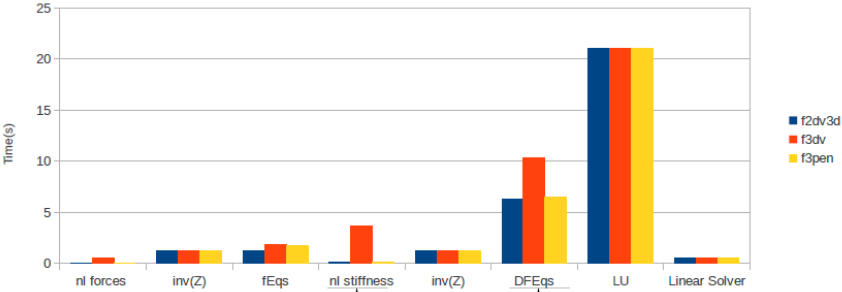
Newton family

1. Newton-Raphson or Newton-Gauss
2. Jacobian Free Krylov-Newton
3. Quasi-Newton methods

fixed point methods

1. Over-relaxation method
2. fixed-point/newton technique
3. Latin Method
4. pseudo-time methods

CPU Time



$$f(\tilde{\mathbf{X}}) = \left\{\tilde{\mathbf{X}}\right\} - [\mathbf{Z}(\omega)]^{-1}\left(\left\{\tilde{\mathbf{F}}_{ex}\right\} - \overbrace{\left\{\tilde{\mathbf{F}}_{NL}\right\}}\right) \quad \mathbf{J}_f = \mathbf{I} - \overbrace{[\mathbf{Z}(\omega)]^{-1}}\frac{D\tilde{\mathbf{F}}_{NL}}{D\tilde{\mathbf{X}}}$$

Linear Solver GMRES

- GMRES is used to find Newton direction:

$$\boldsymbol{D_X f} \Delta p^k = -f(\tilde{X}^k) \rightarrow \boldsymbol{A} u = b$$

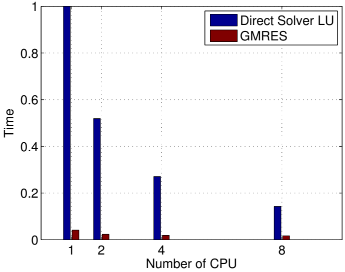
- GMRES is bassed on building of Krylov subspace defined by

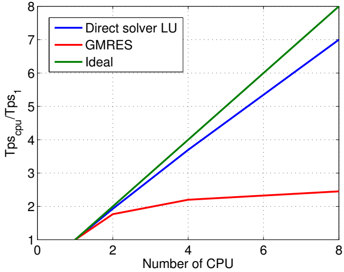
$$\mathcal{K}_n = span\left\{A, Ab, A^2 b, \cdots, A^{n-1} b\right\}$$

- At each iteration of Newton's solver of each frequency GMRES is initialized by the solution of GMRES solver at the previous iteration of Newton solver
- GMRES(m) with restart each m iteration is chosen to limit memory use

Comparison of Linear Solver

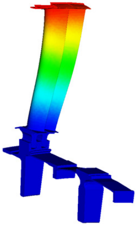
- 16686 non-linear equations
- Parallel coding with OPENMP
- Use of the Intel MKL Library

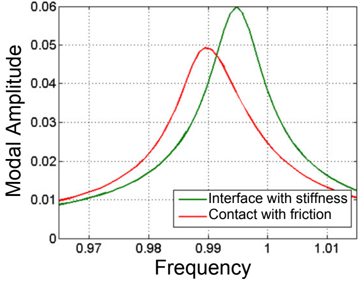


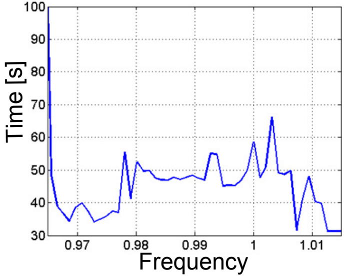


Forced Response of bladed-disk

- First mode of bending 1F
- harmonic order: $0, 1, 3$
- 27810 NL equations
- time: 88 min with 8 cores
- ratio CPU time/wall clock 5.358







Outline

Floquet Theory

Floquet Theorem

Floquet proved that the stability of periodic solutions of a dynamical system defined by

$$\dot{\boldsymbol{x}} = A(t)\boldsymbol{x}$$

can be verified, studying the eigenvalues of the monodromy matrix of this system related to the solution x_s, with $A(t)$ a piecewise continuous periodic function with period T and defines the state of the stability of solutions.

Floquet Theory

Definition of a Monodromy Matrix

- $y(t)$ is a small perturbation of the solution $x_s(t)$: $\dot{\boldsymbol{y}} = \boldsymbol{D_x F y}$
- Monodromy matrix Φ: $\boldsymbol{y}(t+T) = \Phi \boldsymbol{y}(t)$

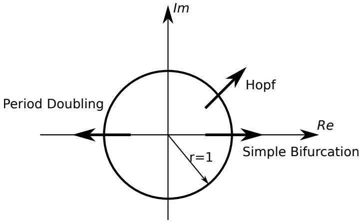


Figure: Bifurcation of unstable periodic solutions

Calculation of Monodromy Matrix

Available methods in time domain

- 2n-pass numerical integration
- Approximation of the matrix exponential
- Runge-Kutta single pass
- Chebyshev polynomials
- Wavelet Galerkin procedure
- Single pass Newmark integration

Available methods in frequency domain

- Hill's method (Fourier series)
- Hill's method with a check of eigenvectors

Comparison of different methods 1/2

Peletan *et al.* - A comparison of stability computational methods for periodic solution of nonlinear problems with application to rotordynamics // Nonlinear Dyn (2013) 72:671682

Rotor model	n_{ele}	n	N	n_{HBM}
Jeffcott v.1	N/A	2	24	98
Jeffcott v.2	N/A	4	32	260
FE rotor v.1	4	24	12	600
FE rotor v.2	6	34	12	850
FE rotor v.3	9	49	12	1225
FE rotor v.4	13	69	12	1725
FE rotor v.5	17	89	12	2225

Comparison of different methods 2/2

Rotor model	n_{ele}	n	N	n_{HBM}
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Rotor model	No stab.	Frequency domain	Time domain			
		Hill2	$2n$-pass	Exponentials	RK 1-pass	Nm 1-pass
Jeffcott v.1	1	9.5*	23	1.7	1.3	1.4
Jeffcott v.2	1	151*	45	2.0	1.6	1.5
FE rotor v.1	1	$2.4 \times 10^{3*}$	1.4×10^3	10	7.9	1.8
FE rotor v.2	1	$4.6 \times 10^{3*}$	5.3×10^3	22	16	2.3
FE rotor v.3	1	$1.1 \times 10^{4*}$	1.3×10^4	51	34	3.5
FE rotor v.4	1	$5.9 \times 10^{4*}$	3.7×10^4	170	81	5.7
FE rotor v.5	1	$1.2 \times 10^{5*}$	9.9×10^4	260	300	10

Single pass Newmark method

Equation of motion of the perturbed system

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{C}(t)\dot{\boldsymbol{y}} + \boldsymbol{K}(t)\boldsymbol{y} = \boldsymbol{0}$$

Transition between time steps

$$\boldsymbol{Y}_{k+1} = \boldsymbol{D}_k \boldsymbol{Y}_k \quad \text{and} \quad \boldsymbol{\Phi} = \prod_k \boldsymbol{D}_k$$

Transition matrix

$$\boldsymbol{D}_k = \boldsymbol{H}_1^{-1} \boldsymbol{H}_0$$

with

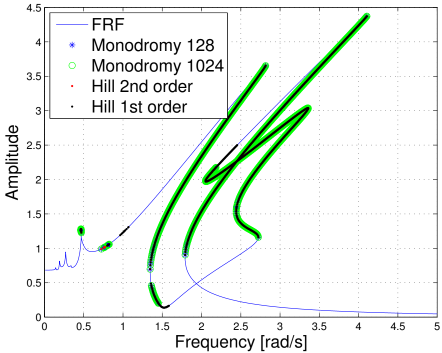
$$\boldsymbol{H1} = \begin{bmatrix} \boldsymbol{M} + \beta h^2 \boldsymbol{K}_{k+1} & \beta h^2 \boldsymbol{C}_{k+1} \\ \gamma h \boldsymbol{K}_{k+1} & \boldsymbol{M} + \gamma h \boldsymbol{C}_{k+1} \end{bmatrix}$$

and

$$\boldsymbol{H0} = \begin{bmatrix} \boldsymbol{M} - (\frac{1}{2} - \beta) h^2 \boldsymbol{K}_k & h\boldsymbol{M} - (\frac{1}{2} - \beta) h^2 \boldsymbol{C}_k \\ -(1-\gamma) h \boldsymbol{K}_k & \boldsymbol{M} - (1-\gamma) h \boldsymbol{C}_k \end{bmatrix}$$

Duffing model

Duffing with gyroscopic element



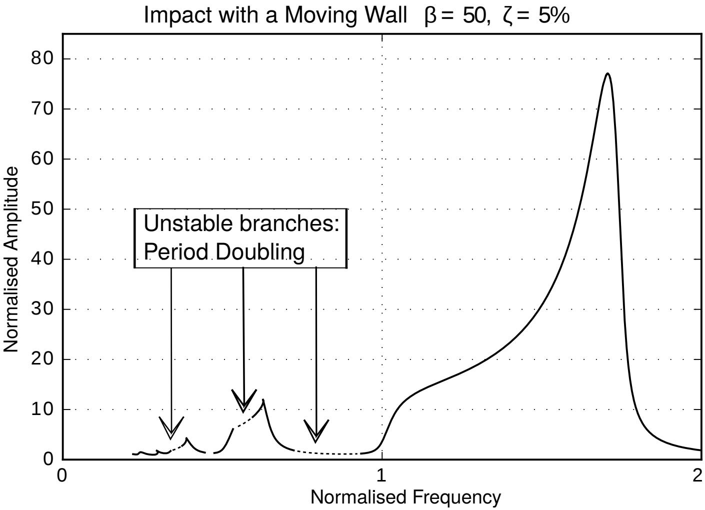
Elements available in FORSE

Available nonlinear elements

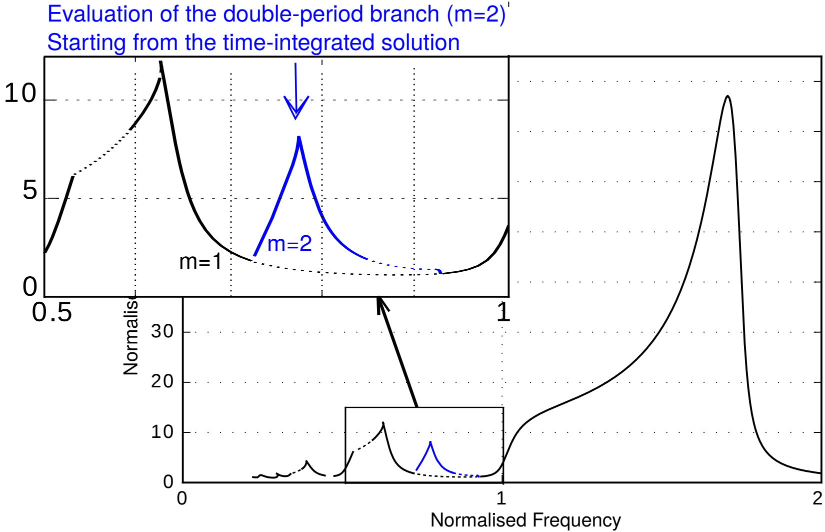
- piecewise linear element
- gap element
- gyroscopic elements
- power law elements
- snubber elements
- viscous damper

Outline

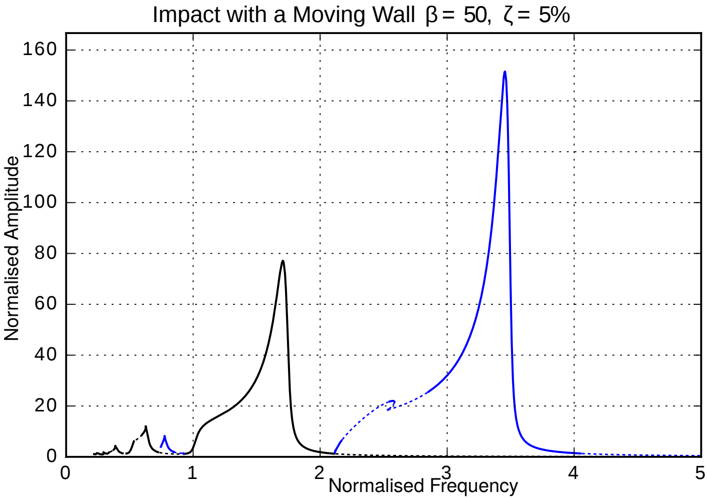
Period Doubling Bifurcation



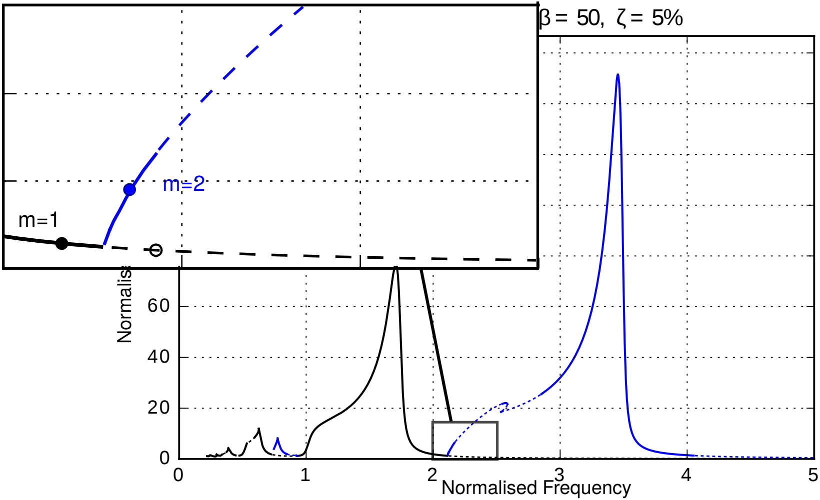
Period Doubling Bifurcation



Period Doubling Bifurcation



Period Doubling Bifurcation



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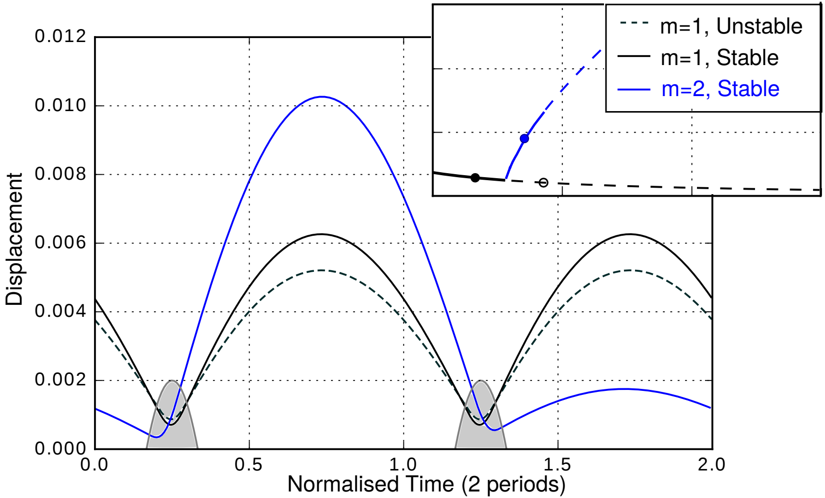
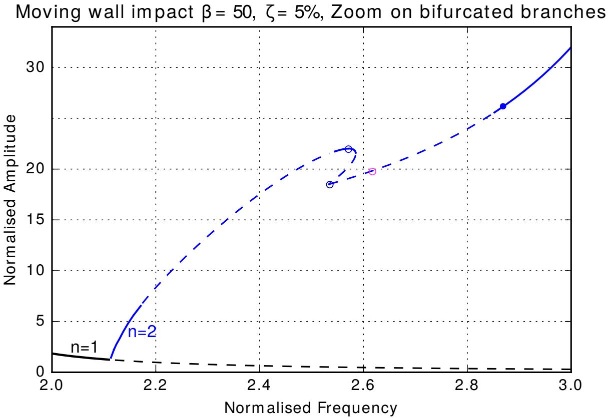


Stability and Bifurcations



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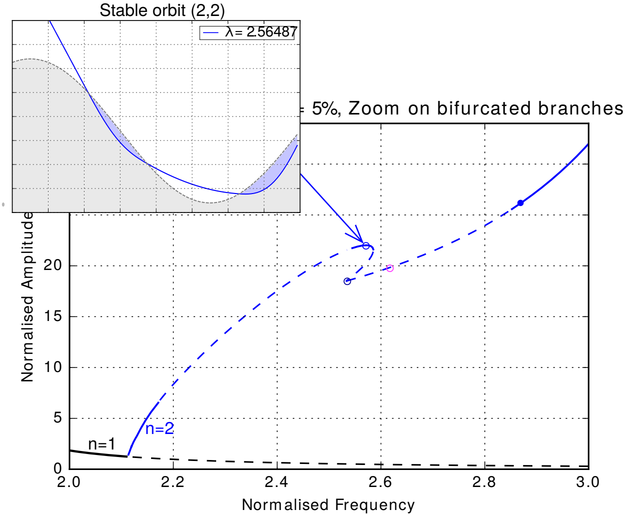

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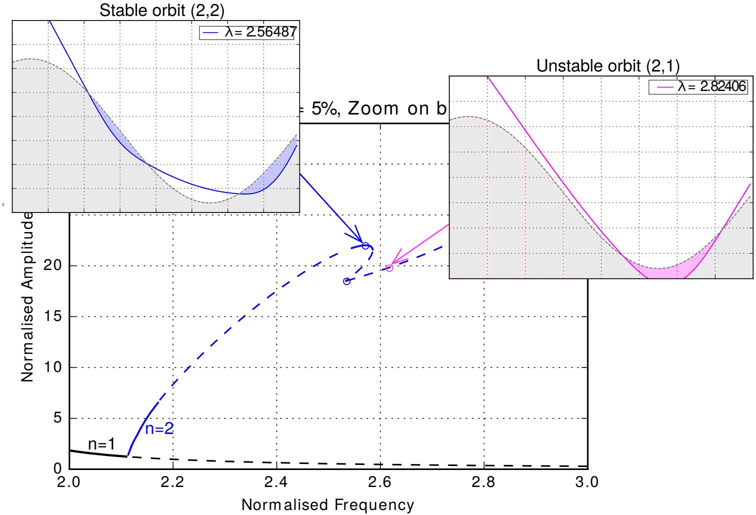

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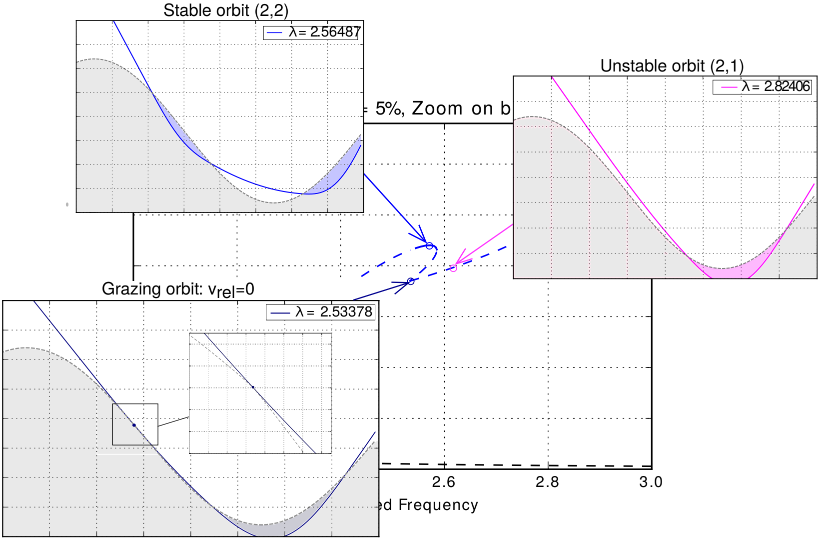
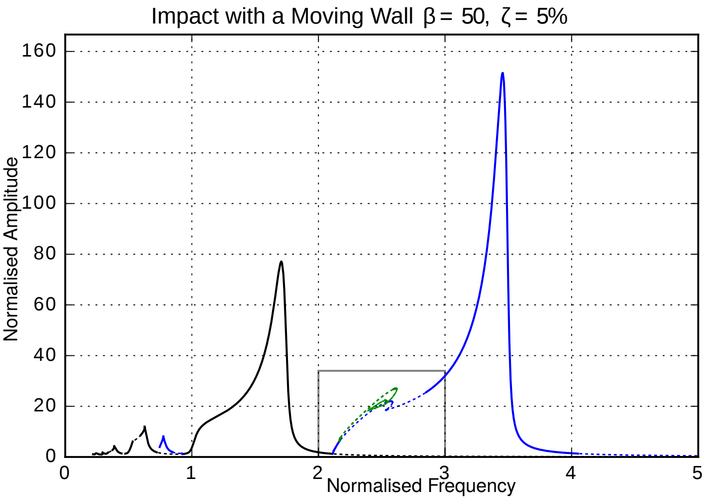
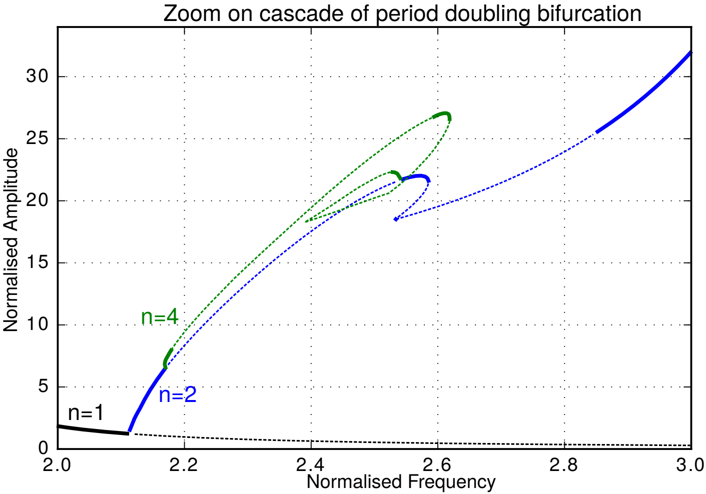


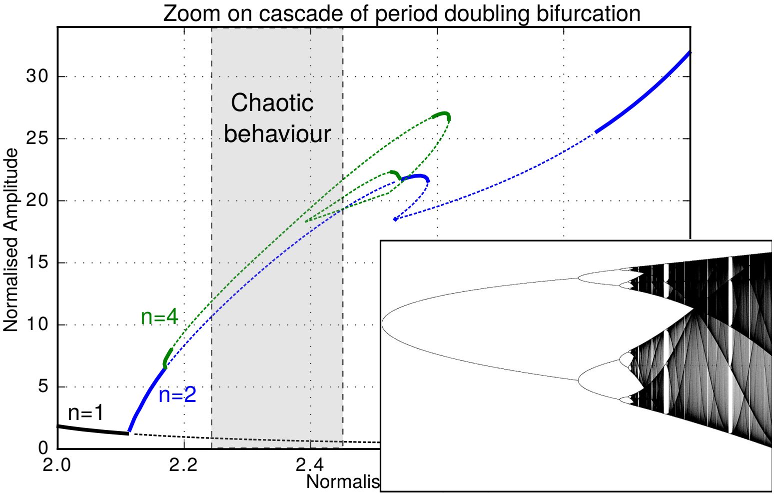
Stability and Bifurcation of the Impacting System



Stability and Bifurcations



Stability and Bifurcations



Outline

Main direction

- Parallel Linear Solver
- Parallelization of HBM/AFT
- Domain Decompositon Methods with ROM
- Non-linear localization technique with Schur complement technique
- Parallel Preconditionners
- Optimization of the Code
- New architecture: GPU, Vectorization...

Reference

References

Malte, K. Salles, L. and Thouverez F. "Vibration prediction of bladed disks coupled by friction joints." Archives of Computational Methods in Engineering. 2017 24(3)

Cardona A.L., Lerusse A.L., Gradin M.I. Fast Fourier nonlinear vibration analysis. Computational Mechanics. 1998 22(2)

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Seydel, R. Practical Bifurcation and Stability Analysis, 2009

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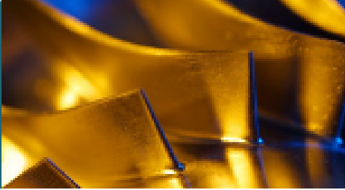
Librairies

- LOCA `https://trilinos.org/packages/nox-and-loca/`
- Mdultifario `http://multifario.sourceforge.net/`
- Matcont `https://sourceforge.net/projects/matcont/`
- PyDSTool `http://www2.gsu.edu/~matrhc/PyDSTool.htm`



Introduction to Continuation Methods

Loïc Salles

10 January 2018